The local mortician charges by the pound for embalming according to the following table:

Weight	**Cost**
For the first 50 lbs	5.00 per pound
For each additional pound over 50 lbs	2.00 per pound

Find a piecewise linear function that models the cost as a function of weight.

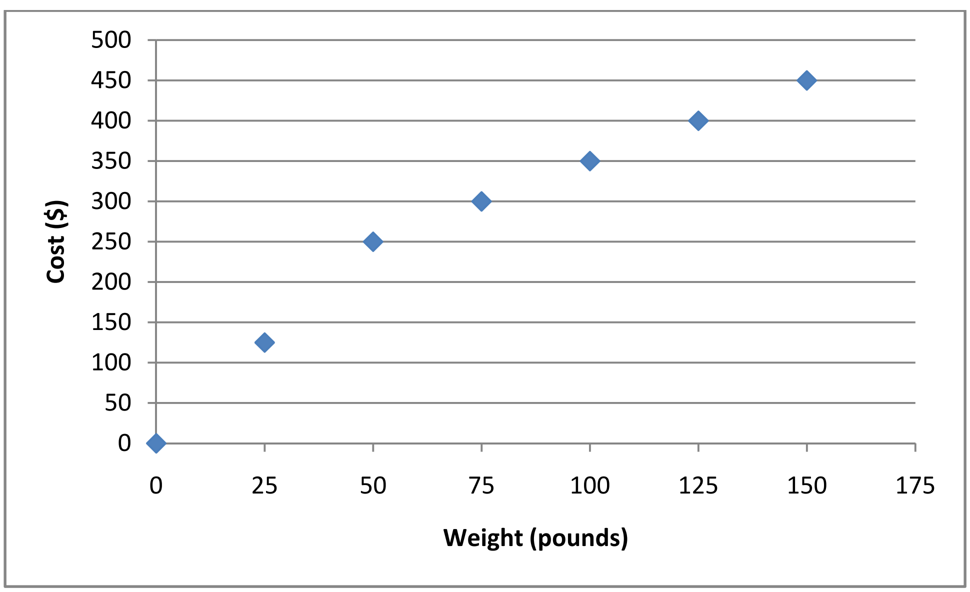
To get started on this problem, it is best to have some visual representation of the function so we can see why a piecewise linear function is appropriate. To help us make this graph, let's make up a table of values and find some corresponding costs.

Weight in pounds	Cost in dollars
0	0
25	125
50	250
75	300
100	350
125	400
150	450

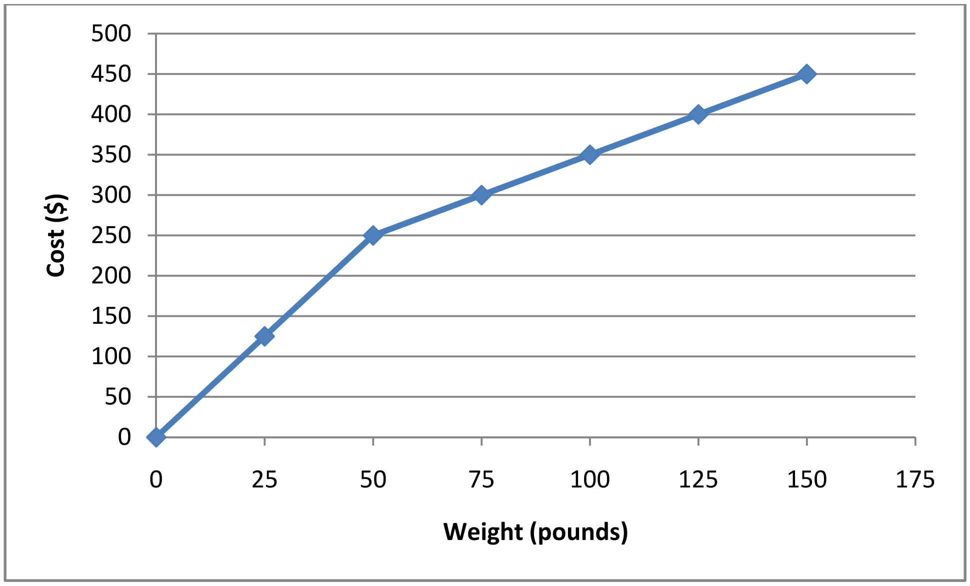
$$(25\text{ lbs})\left(5\frac{\text{dollars}}{\text{lb}}\right) = \$125$$

$$(50\text{ lbs})\left(5\tfrac{\text{dollars}}{\text{lb}}\right) + (25\text{ additional lbs})\left(2\tfrac{\text{dollars}}{\text{lb}}\right) = \$300$$

Now let's make a graph of these data points.



Examining the data and the shape in this graph, you might make an educated guess that the graph of the function might look like



The graph seems to consist of two pieces. One piece extends from 0 pounds to 50 pounds and the other extends beyond 50 pounds (which makes sense with the description given). Both pieces are straight lines, but each has a different formula. Let's look at each piece and deduce the formula for that piece.

Weights from 0 to 50 pounds

This piece passes through three points on the graph: (0, 0), (25, 125) and (50, 250). To find the equation of this segment, we'll use the slope intercept form of a line $y = mx + b$. In this case, we'll change the variable names to make it more appropriate. Let's write

$$c = mw + b$$

were c is the cost to embalm a person weighing w pounds.

Since (0, 0) is the y-intercept, we know that the value of b must be 0. We can calculate the slope from any pair of the three points. If I use the second and third ordered pair, I get

$$m = \frac{250 - 125}{50 - 25} = 5 \frac{\text{dollars}}{\text{lb}}$$

To write out the equation, I set m = 5 and b = 0 to get

$$c = 5w \quad \text{for } 0 \le w \le 50$$

The inequalities on the end let us know that this formula is only valid for weights from 0 to 50 pounds.

Weights above 50 pounds

The same approach we used above will also work in this section. Let's pick two points from that piece and use $c = mw + b$ to find the formula. Any pair of points in the second piece will work, but let's use (75, 300) and (150, 450). Using these points, the slope is calculated to be

$$m = \frac{450 - 300}{150 - 75} = 2\,\frac{\text{dollars}}{\text{lb}}$$

This means that we can write our equation as

$$c = 2w + b$$

This segment of the function does not cross the y axis, so to find b we'll need to substitute one of our points into the function. Pick (75, 300) and set w = 75 and c = 300:

$$300 = 2(75) + b$$

Simplifying this and solving for b gives us

$$\begin{aligned} 300 &= 150 + b \\ 150 &= b \end{aligned}$$

This means that the equation for this segment is

$$c = 2w + 150 \quad \text{for } w > 50$$

Put It All Together

Let's look at where we are. We have the formula for each piece:

$$c = 5w \quad \text{for } 0 \le w \le 50$$

$$c = 2w + 150 \quad \text{for } w > 50$$

In a picewise function, we write these pieces together. Let's call this function C(w), the cost to embalm a body weighing w pounds.

$$C(w) = \begin{cases} 5w & \text{for } 0 \le \text{w} \le 50 \\ 2w + 150 & \text{for } w > 50 \end{cases}$$

This formulation takes the two pieces and puts them together into a single function C(w).

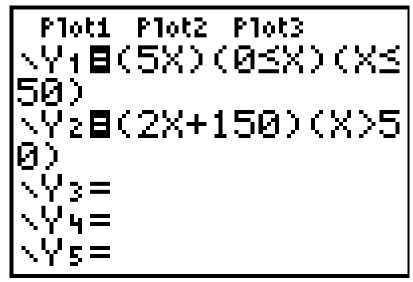
Graph on a TI-83

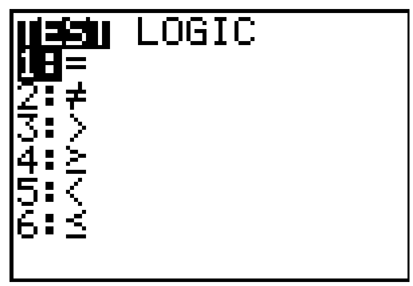
Finally, let’s graph this function

$$C(w) = \begin{cases} 5w & \text{for } 0 \le \text{w} \le 50 \\ 2w+150 & \text{for } w > 50 \end{cases}$$

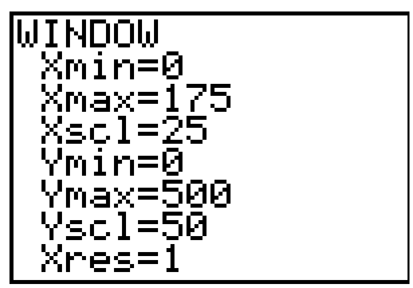
and make sure it matches the data points we made a scatter plot of earlier.

1. Press [Y=] to enter the equation editor.
2. Enter the formula shown on the first line in Y_1. To enter the inequalities, you’ll need to go to the TEST menu by typing [2nd][MATH]. Notice that you’ll need to write to sets of inequalities for $0 \le w \le 50$ since this is equivalent to $0 \le w$ and $w \le 150$.
3. Enter the formula shown on the second line in Y_2. As before use [2nd][MATH] to put in the inequalities.

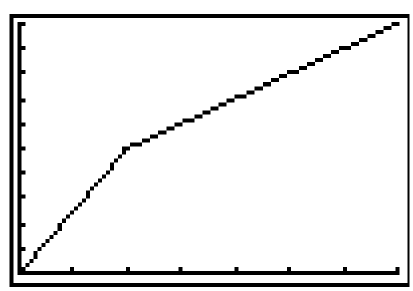




4. To adjust the viewing window, press [WINDOW].
5. Enter the values corresponding to our earlier scatter plot.



6. Press [GRAPH] to see the piecewise linear function.



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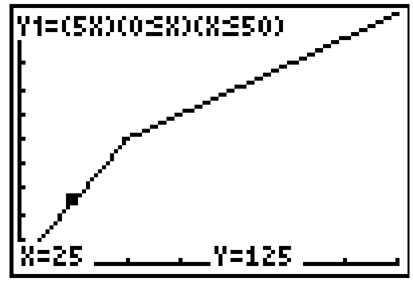
<td>7. To check to see if this is correct, press [TRACE]. Jump to the Y_1 formula using the cursor control keys [▲] or [▼] if needed.
8. Type [2][5][ENTER] to go to the point corresponding to a weight of 25 pounds. Note that the value matches with the table we created earlier.</td>
<td>
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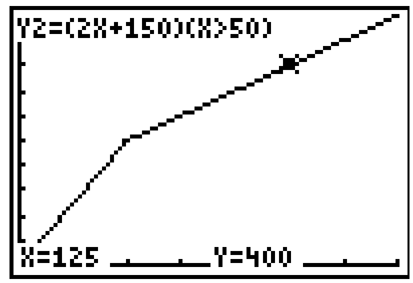
<td>9. Move to the Y_2 part of the function using [▲] or [▼].
10. Type [1][2][5][ENTER] to go to the point corresponding to a weight of 125 pounds. Note that the value matches with the table we created earlier.</td>
<td>
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